OPTIMAL TASK ASSIGNMENT IN SENSOR NETWORKS

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MOTIVATION

- In-network processing paradigm
- Millions of sensors with processing capabilities
- Vision
  - Reduce the volume of data seen / processed by the application
- Complex application tasks can split into simpler tasks that can be accomplished by separate nodes within the network
- Goal: assign the tasks optimally in terms of energy
  - Sensing
  - Processing
  - Communication
  - ...

Optimal Task Assignment in Sensor Networks
PROBLEM DEFINITION

- Definition (Optimal Assignment)

Let $W$ be a WSN and let $T$ be a complex task. Given the set of all mappings $\Phi = \{\phi : T \rightarrow W\}$ where the capabilities of the network fulfill the task and energy requirements, we call an assignment as energy-optimal assignment $\phi_{opt}$ if and only if

$$\left( \forall \phi \in \Phi \right) E_{W,T}^{\phi_{opt}} \leq E_{W,T}^{\phi}.$$ 

- NP-hard [Garey and Johnson, 1979]
**MODELING FRAMEWORK**

**Task Model**

**Definition (Task).** Given a set $S$ of subtask vertices and a set $C$ of directed edges among the subtask vertices, we define a task $T$ as a *directed acyclic graph (DAG)* represented by the tuple $T \equiv \langle S, C \rangle$.

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Input From</th>
<th>Output To</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensing</td>
<td>Sense context from the environment</td>
<td>-</td>
<td>• Processing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Sink</td>
</tr>
<tr>
<td>Processing</td>
<td>Process input data and forward results</td>
<td>• Processing</td>
<td>• Processing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Sensing</td>
<td>• Sink</td>
</tr>
<tr>
<td>Sink</td>
<td>Retrieve final format of data sensed and processed</td>
<td>• Processing</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Sensing</td>
<td></td>
</tr>
</tbody>
</table>
MODELING FRAMEWORK
Task Model - Example

\[ p = \langle \text{sink}, 1, \Sigma \rangle \]

\[ q = \langle \text{max}_{100}, 1, \Sigma \rangle \]

\[ r = \langle \text{temperature}, 1000, \Xi_1 \rangle \]

\[ s = \langle \text{temperature}, 1000, \Xi_2 \rangle \]

\[ C_{qp} = \langle q, p, 1600 \rangle \]

\[ C_{rq} = \langle r, q, 16000 \rangle \]

\[ C_{sq} = \langle s, q, 16000 \rangle \]
Definition (Network). Let \( N \) be a set of network nodes and let \( A \) be a set of directed network edges. We define a WSN as a *strongly connected directed graph* represented by the tuple \( W \equiv \langle N, A \rangle \).
MODELING FRAMEWORK

Network Model - Example

\[a = \{\{\text{humidity, max}\}, \bar{x}_4, 38000, \{M_a (\text{humidity}) = 0.001, M_a (\text{max}) = 0.01\}\}\]

\[b = \{\{\text{temperature, max}\}, \bar{x}_3, 50000, \{M_b (\text{temperature}) = 0.002, M_b (\text{max}) = 0.005\}\}\]

\[c = \{\{\text{temperature, max}\}, \bar{x}_1, 10800, \{M_c (\text{temperature}) = 0.001, M_c (\text{max}) = 0.01\}\}\]

\[d = \{\{\text{sink, temperature}\}, \bar{x}_2, \infty, \{M_d (\text{sink}) = 0, M_d (\text{temperature}) = 0.002\}\}\]

\[A_{bd} = \{b, d, 0.9, 0.8\}\]

\[A_{ad} = \{a, d, 0.6, 0.8\}\]

\[A_{da} = \{d, a, 0.8, 0.7\}\]

\[A_{ac} = \{a, c, 0.2, 0.2\}\]

\[A_{ab} = \{a, b, 0.2, 0.1\}\]

\[A_{ca} = \{c, a, 0.2, 0.1\}\]

\[A_{ba} = \{b, a, 0.2, 0.1\}\]
MODELING FRAMEWORK
Assignment Model

Definition (Assignment). Let \( T \equiv \langle S, C \rangle \) be a complex task and \( W \equiv \langle N, A \rangle \) be a WSN with \( \Pi = \{ \Pi_{ab} \} \), \( \forall a, b \in N \) representing the set of all paths among the network nodes. We define the assignment \( \zeta : T \to W \) as the pair of mappings \( X_\zeta : S \to N \) and \( Y_\zeta : C \to \Pi \) that satisfy the consistency constraint:

\[
\left( \forall C_{pq} \in C \right) \left( \forall \Pi_{ab} \in \Pi \right) \quad Y_\zeta \left( C_{pq} \right) = \Pi_{ab} \Leftrightarrow \left( X_\zeta \left( p \right) = \text{Src} \left( \Pi_{ab} \right) \right) \land \left( X_\zeta \left( q \right) = \text{Dst} \left( \Pi_{ab} \right) \right)
\]

and we represent it by \( \zeta \equiv \langle X_\zeta, Y_\zeta \rangle \in Z \) where \( Z \) is the set of all possible assignments between \( T \) and \( W \)
MODELING FRAMEWORK
Assignment Model - Example

\[ X_\zeta = \{ p \rightarrow d, q \rightarrow a, r \rightarrow c, s \rightarrow b \} \]
\[ Y_\zeta = \{ C_{qp} \rightarrow \Pi_{ad}, C_{rq} \rightarrow \Pi_{ca}, C_{sq} \rightarrow \Pi_{ba} \} \]

Optimal Task Assignment in Sensor Networks
### OPTIMIZATION

#### ILP Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type</th>
<th>Size</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{p,a}$</td>
<td>Binary Matrix</td>
<td>$</td>
<td>S</td>
</tr>
<tr>
<td>$y_{pq,ab}$</td>
<td>Binary Matrix</td>
<td>$(</td>
<td>S</td>
</tr>
<tr>
<td>$\beta_{pq}$</td>
<td>Integer Column</td>
<td>$</td>
<td>S</td>
</tr>
<tr>
<td>$\rho_{ab}$</td>
<td>Real Column</td>
<td>$</td>
<td>N</td>
</tr>
<tr>
<td>$\rho_{ab,c}$</td>
<td>Real Matrix</td>
<td>$</td>
<td>N</td>
</tr>
<tr>
<td>$\varepsilon_{c,p}$</td>
<td>Real Column</td>
<td>$</td>
<td>S</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Real Column</td>
<td>$</td>
<td>N</td>
</tr>
</tbody>
</table>
**ALGORITHM 1. ILP Formulation**

**Input:** Sets $C, S, N, \Pi, I_p, J_{pq}$, Variables $\beta_{pq}, \rho_{ab}, \rho_{ab,c}, \varepsilon_{c,p}, \varepsilon_c$

**Output:** Variables $x_{p,a}, y_{pq,ab}$

**Objective function:** minimize \[ \sum_{c \in N} \left( \sum_{p \in S} \varepsilon_{c,p} \cdot x_{p,c} + \sum_{C_{pq} \in C} \sum_{\Pi_{ab} \in \Pi} \beta_{pq} \cdot y_{pq,ab} \cdot \rho_{ab,c} \right) \]

**Constraints set:**
- $\forall p \in S \sum_{a \in N} x_{p,a} = 1$  //Unique subtask vertex assignment constraint
- $\forall C_{pq} \in C \sum_{\Pi_{ab} \in \Pi} y_{pq,ab} = 1$  //Unique subtask edge assignment constraint
- $\forall C_{pq} \in C \forall a \in N \sum_{\Pi_{ab} \in \Pi \cap \sigma(a)} y_{pq,ab} = x_{p,a}$  //Edge-to-source-vertex consistency constraint
- $\forall C_{pq} \in C \forall b \in N \sum_{\Pi_{ab} \in \Pi \cap \pi(b)} y_{pq,ab} = x_{q,b}$  //Edge-to-destination-vertex consistency constraint
- $\forall c \in N \sum_{p \in S} \varepsilon_{c,p} \cdot x_{p,c} + \left\{ \sum_{C_{pq} \in C} \sum_{\Pi_{ab} \in \Pi} \beta_{pq} \cdot y_{pq,ab} \cdot \rho_{ab,c} \right\} \leq \varepsilon_c$  //Node energy conservation constraint
- $\forall p \in S \forall a \in N \setminus I_p \quad x_{p,a} = 0$  //Vertex compatibility constraint
- $\forall C_{pq} \in C \forall \Pi_{ab} \in \Pi \setminus J_{pq} \quad y_{pq,ab} = 0$  //Edge compatibility constraint
## Experimental setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>C_l</td>
</tr>
<tr>
<td>(</td>
<td>A_{lg}</td>
</tr>
<tr>
<td>(</td>
<td>S_{ens}))</td>
</tr>
<tr>
<td>(</td>
<td>A_{lg}))</td>
</tr>
<tr>
<td>(Quant))</td>
<td>6000</td>
</tr>
<tr>
<td>(#SinkSubtasks)</td>
<td>1</td>
</tr>
<tr>
<td>(\beta_{pq}))</td>
<td>320 Kbit</td>
</tr>
</tbody>
</table>

*Spatial constraint*: 1:5 sensing subtasks

## Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>S_{ens}))</td>
</tr>
<tr>
<td>(</td>
<td>A_{lg}))</td>
</tr>
<tr>
<td>(#SinkNodes)</td>
<td>1</td>
</tr>
<tr>
<td>(M_{aver}(op,)_)</td>
<td>0.005mJ</td>
</tr>
<tr>
<td>(\rho_{tx,aver} = \rho_{rx,aver} = \rho_{aver})</td>
<td>0.0001mJ/bit</td>
</tr>
<tr>
<td>(\epsilon_{aver})</td>
<td>300J</td>
</tr>
</tbody>
</table>

Optimal Task Assignment in Sensor Networks
# Results

Energy status of the network after Run #100

<table>
<thead>
<tr>
<th>Task</th>
<th>Node A</th>
<th>Node B</th>
<th>Node C</th>
<th>Node D</th>
<th>Node E</th>
<th>Node F</th>
<th>Node G</th>
<th>Node H</th>
<th>Node I</th>
<th>Node J</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>50.2%</td>
<td>60.3%</td>
<td>70.4%</td>
<td>80.5%</td>
<td>90.6%</td>
<td>100%</td>
<td>110%</td>
<td>120%</td>
<td>130%</td>
<td>140%</td>
</tr>
<tr>
<td>T2</td>
<td>40.2%</td>
<td>50.3%</td>
<td>60.4%</td>
<td>70.5%</td>
<td>80.6%</td>
<td>90.7%</td>
<td>100%</td>
<td>110%</td>
<td>120%</td>
<td>130%</td>
</tr>
<tr>
<td>T3</td>
<td>30.2%</td>
<td>40.3%</td>
<td>50.4%</td>
<td>60.5%</td>
<td>70.6%</td>
<td>80.7%</td>
<td>90.8%</td>
<td>100%</td>
<td>110%</td>
<td>120%</td>
</tr>
</tbody>
</table>

Optimal Task Assignment in Sensor Networks
RESULTS

Optimal assignment – Run #1

Optimal Task Assignment in Sensor Networks
RESULTS

Optimal assignment – Run #21

Optimal Task Assignment in Sensor Networks
RESULTS

Mesh network

Optimal Task Assignment in Sensor Networks
RESULTS

Mesh network

Per Alg - ILP - Det. Execution Time

Optimal Task Assignment in Sensor Networks
RESULTS
Mesh network

Per Alg - ILP - Det. Cumulative Execution Time

Optimal Task Assignment in Sensor Networks
FUTURE WORK & EXTENSIONS

- Experimentation with non-optimal solutions that scale
- Maximize the network lifetime instead of minimizing the energy at each step separately
- Model the unreliability of wireless channels
- Insert the concept of mobility (Levy Walk model)
- Incremental assignment of tasks
REFERENCES


