

Automatic Fuzzy Rules Generation for the Deadline Calculation of a Seller Agent

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Abstract—Intelligent Agents can help users in finding and retrieving goods from electronic marketplaces. Additionally, agents can represent providers in such places facilitating the automatic negotiation about the purchase of products. In this paper, we describe a finite horizon bargaining model between buyers and sellers and we focus on the seller's side. Seller agents are a good example of an autonomous decentralized system. We present a method for the 'bargaining' deadline calculation based on Fuzzy-Logic (FL). Such deadline indicates the time for which it is profitable for a seller to participate in the bargaining procedure. We provide methods for automatic fuzzy rules generation. These rules result the deadline values at each interaction and are based on data provided by experts. We compare results taken from a Fuzzy controller based on such automatic methods with results taken by previous research efforts.

Keywords-Electronic Markets, Bargaining Games, Fuzzy Logic.

I. INTRODUCTION

Today, users are in front of a huge amount of sources where they can discover and retrieve what ever they need to buy. Users need an automatic way to find products in the Web. Intelligent autonomous software components, such as agents, seem to be the appropriate solution to this problem. Agents are capable of acting autonomously in order to achieve goals defined by their owners. Hence, agents can undertake the responsibility of finding products in the Web with the minimum users intervention.

The usage of agents in Electronic Markets (EMs) could be highly advantageous for the product discovery and acquisition. Agents can represent users and providers in an EM, thus, facilitating the automatic negotiation about the purchase of products. We study a buyer-seller interaction model. Our model enables the engineering of algorithms and protocols for more efficient transactions. This model is based on Game Theory (GT) [1]. GT provides an efficient way to describe interactions between entities that try to maximize their profits.

Fuzzy Logic (FL) can enhance the interaction between these entities. It is an algebra based on fuzzy sets [12] providing approximate reasoning mechanisms. FL deals with incomplete or uncertain information and helps at representing the knowledge of agents involved in an EM. Hence, agents can automatically decide during the interaction. An important decision is the calculation of the correct time for which an

agent should participate in the interaction procedure. The calculation of that deadline affects the behaviour of the seller concerning the proposed prices. We use FL for representing the seller knowledge and for calculating the appropriate deadline in a bargaining game. The product's popularity and the initial intended profit consist of the necessary information for the deadline calculation. We study and compare models that are based on the automatic rule base extraction from data given by experts. In most of fuzzy systems, human experts define and tune the fuzzy rule base. This requires time, experience and skills in order to have an efficient fuzzy rule base. The most important is that probably such rules will not be optimal leading to non-productive results. Our work extends the work presented in [10] and [11] and we compare our results with results taken by a model where experts define the fuzzy rule base.

The rest of this paper is organized as follows: Section II reports prior work while in Section III we give the necessary description of EMs and the analysis of the method followed by agents buying / selling products. Such method is a Bargaining Game (BG) [2]. We pay special attention on the behaviour of the seller and we describe its basic characteristics. Section IV is devoted to the description of the fuzzy controller that results the appropriate deadline for the seller according to the values of the products popularity and the intended profit. The deadline represents the time interval for which the BG is efficient from the seller's viewpoint. Moreover, we described the fuzzy rule base and the methodology followed for the automatic generation of rules. In Sections V and VI, we conclude the paper by presenting our key findings.

II. PRIOR WORK

Nowadays, one can find some very interesting efforts in the design and development of virtual marketplaces. A key pursuit in such architectures is to define effective mechanisms for automatic negotiation between market participants.

Authors in [3] present a BG, which is held between buyers and sellers, describing a set of strategies for both sides. They describe symmetric and asymmetric scenarios concerning the knowledge of the opponent's parameters. The examined parameters are the players' deadline, reservation prices and discount factors. A theoretical model regarding players deadline is presented in [4]. The BG in [4] is between agents and the sequential equilibrium is studied. Authors formulate

the game between two agents, which are of different types and have their own deadlines. The involved parties do not share knowledge (i.e. type of the opponent and deadline).

In [5] and [6], the authors adopt FL in agent systems. They describe reasoning mechanisms used for agents' negotiations. Specifically, authors in [5] present a sequential bargaining technique based on FL for the estimation of acceptable prices of parties trying to form joint ventures (JV) of companies. In [6], authors present the rationale of an intelligent fuzzy-based agent negotiating in an e-commerce environment. The inference rules and the decision strategies are described along with the relevant implementation.

The use of FL in Continuous Double Auctions (CDAs) is studied in [7]. The scenario involves the buyer and the seller. Authors present the algorithms used by agents participating in such places and employ a number of heuristic fuzzy rules and fuzzy reasoning mechanisms in order to determine the optimum bid for specific products.

In [8] agents use a decision function at every round of a BG. This function deals with current and last proposals by the two negotiating agents. Authors define three different strategies for the counter proposals definition: time-dependent, behaviour-dependent and resource-dependent. In [9], authors define functions for the definition of the proposals in alternating offers interaction and describe a set of tactics. Both efforts ([8] & [9]) deal with deadlines, however, they do not provide means for their calculation. Moreover, they do not take into consideration the fact that agents can operate under pressure which is imposed by time limitations or the incomplete knowledge about the opponent's characteristics.

In [10] a bargaining scenario for agents participating in Information marketplaces is presented. The direct interaction between buyers and sellers is studied. This interaction involves a set of alternating offers for a specific product. Authors describe a mathematical model for the seller's deadline calculation. Afterwards, authors in [11] describe a fuzzy model for the deadline calculation. A set of fuzzy rules are defined according to experts' knowledge for the discussed scenario.

In this paper, we try to define a model that imitates human behaviour for negotiations held in marketplaces. Previous efforts describe that the participants have time limitations, however, they are not deal with them. Especially, in the seller's side and in contrast to other efforts, we present a simple mathematical approach for the deadline calculation and extend it using a Fuzzy controller. The important is that though clustering, we indicate the procedure for the automatic rules definition which finally results the deadline. Hence, the seller is based on simple numbers defined by experts and thus saves time because it does not need to be based on the definition of specific rules. Our approach is characterized by simplicity because it does not require any special effort for the definition of the fuzzy rule base. Moreover, the definition of specific If – Then rules will probably not be efficient in all of the cases that the agent will face in a dynamic environment. Hence, we present a way to have autonomous software components that dynamically decide their actions using a fuzzy rule base which can be created by clustering simple numbers.

III. BARGAINING SETUP

Electronic marketplaces can be considered as places where entities not known in advance can negotiate and agree upon the exchange of products. In such places two groups are the basic players: buyers and sellers. Buyers are entities seeking for information products while sellers have a number of products in their property and try to sell them in the most profitable price. All of these entities can be represented by intelligent agents.

We can focus on the direct interaction between buyers and sellers. We model this interaction as a finite-horizon BG under incomplete information reported in [1]. An entity involved in the BG does not have any knowledge about the characteristics of its opponent. The BG lasts for a finite period of time (horizon) and involves a number of alternating offers. At every round, entities propose a specific price for the product. If this price is accepted by the opponent then the BG ends with an agreement and specific profit for both entities. The seller starts first and the buyer follows. If a player is not satisfied with the proposed offer, it has the right to reject it and issue a counter-proposal. In the case of an agreement, the BG ends with profit for both, or else, a "conflict" is experienced leading to zero profit for both involved parties.

We note that, there is a specific time horizon for the BG. The buyer has a specific deadline posed by its owner, while the seller calculates its deadline, as discussed below. If one of the deadlines expires and no agreement is reached then the BG ends with a conflict.

A. Seller Behavior

In this section, we briefly describe the behaviour of the seller (a more extensive discussion can be found in [10]). The seller retrieves products from sources and behaves like a caching server, in the sense that, it can deliver the product to interested parties more than once. The goods, available at the seller, are ranked according to their popularity. The seller has a certain utility function that reflects the attained profit according to the product's price and the seller's cost. An increase in the price of a product triggers an increase in the seller's profit. In the worst case, the seller's profit is equal to zero. At every even BG round, the seller receives the buyer's offer and proceeds as follows:

- If the deadline of the seller, the price proposed by the buyer and, the estimate of the buyer's deadline indicate that, the BG is to carry on then the seller rejects the buyer's offer and issues a new proposal.
- Otherwise, the seller fears a sudden termination of the BG and accepts the current offer.

The buyer does not know the seller's characteristics. The seller characteristics include: the cost (c), the discount factor (δ_s), the utility function (U_s) and the deadline for non-zero profit (T_s). The discount factor indicates that, the seller loses profit as the BG keeps on. Furthermore, the seller knows the popularity of the product, which affects its pricing policy. This means that, it is possible to sell products of high popularity in smaller prices than other products with low popularity.

Popular products yield an added value to their owner since they are sold numerous times to interested buyers. In every odd round, the seller proposes prices using a pricing function which is:

$$p^s(x) = \frac{\varepsilon}{x^{q+1}} + c, \quad x = 1, 2, \dots \quad (1)$$

The term $p^s(x)$ denotes the seller's price at the seller's round x (i.e., $x=2$ implies the second proposal from the seller issued at the third BG round), ε denotes the profit, c is the product's cost and q is the popularity measure (non-normalized probability of reference) for item i according to Zipf's law [13]. It stands:

$$q = i^{-\lambda} \quad (2)$$

where i is product's popularity ranking, λ is the Zipf parameter.

Based on the analysis in [10], we can conclude that, there is a time limit, T_s , beyond which all proposed prices by the seller change marginally. Hence, from its point of view, if the buyer has not accepted its prices up to this time limit, it is aimless for the seller to continue the BG. This is because the policy of the buyer is to wait for the next rounds in order to achieve higher profit. For this reason, the seller defines this time limit which is calculated as:

$$t \cong (\alpha \cdot \varepsilon \cdot (q + 1))^{\frac{1}{q+2}} \quad (3)$$

The parameter α , in Equation (3), is a scaling factor, which depends on the seller's policy; specifically, if the seller follows a patience policy, the α factor assumes a high value. For example, the value of $\alpha=100$ shows that the seller slowly reaches its lower cost bound, in contrast to a value of $\alpha=20$. Smaller values for α indicate an impatient seller that proposes a few prices and concludes rapidly the BG. Equation (3) indicates from which time instant the seller converges to a price that has marginal differences with all the upcoming offers. Specifically, the offers are close enough to the seller's cost and it is meaningless for the seller to continue the BG if the buyer, up to the current time instance, has rejected all the preceding offers.

IV. FUZZY LOGIC CONTROLLER

In this work, our aim is to find an efficient way to define the appropriate value of parameter α and, thus, to define the appropriate deadline for the seller. FL seems to be the appropriate tool for such scenarios helping at decision making under uncertainty. We exploit FL in order to apply a fuzzy rule-based system capable of making decisions of the seller to the product's characteristics and current value of profit. The knowledge of the seller is described by a set of fuzzy rules which define the deadline at every transaction. In Figure 1, we can see the FL controller architecture for the deadline calculation. The seller passes the product's rank and the profit

to the controller which fuzzifies them and passes them to the inference engine. The inference mechanism is based on a fuzzy rule base retrieving the result which is defuzzified in order to produce the final value for parameter α and consequently the deadline.

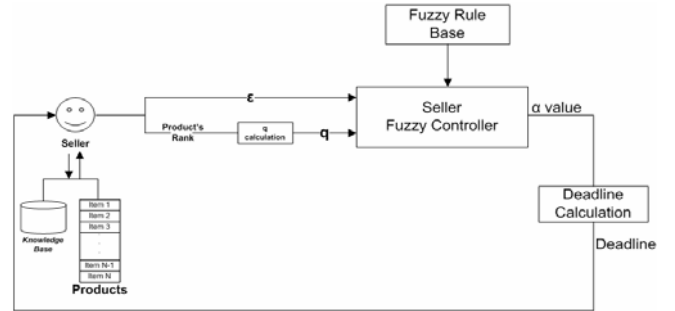


Figure 1. FL controller for the deadline calculation.

The seller conserves a knowledge base in which there is the necessary information for the determination of the above described parameters. At the beginning of each interaction, the seller retrieves from its knowledge base the initial intended profit and the product's ranking. These values are used by the controller in order to determine the value of the parameter α and accordingly the value of the deadline.

A. Fuzzy Rule Base

As we saw in the previous Section, the inference procedure in the fuzzy controller incorporates a rule-based approach. There are two main models of Fuzzy Systems. The Mamdani model [14] utilizes rules as the following:

$$R_j: \text{IF } x_{1j} \text{ is } A_{1j} \text{ AND/OR } x_{2j} \text{ is } A_{2j} \text{ AND/OR } \dots \text{ AND/OR } x_{nj} \text{ is } A_{nj} \text{ THEN } y_j \text{ is } B_j$$

where R_j is the j^{th} Fuzzy rule, x_{ij} ($i=1 \dots n$) is the inputs of the j^{th} rule, y_j is the output of the j^{th} rule and A_{ij} and B_j are membership functions usually associated by linguistic terms. Takagi-Sugeno model [15] involves the following form of rules:

$$R_j: \text{IF } x_{1j} \text{ is } A_{1j} \text{ AND/OR } x_{2j} \text{ is } A_{2j} \text{ AND/OR } \dots \text{ AND/OR } x_{nj} \text{ is } A_{nj} \text{ THEN } y_j = a_{0j} + a_{1j}x_{1j} + a_{2j}x_{2j} + \dots + a_{nj}x_{nj}$$

In this form, each rule has fuzzy antecedents and consequents being linear combinations of inputs. Such models are capable to allow easier application of learning techniques for their identification from data [16]. In our work, we use clustering techniques for the automatic Takagi-Sugeno like rules generation through data given by experts. This is held when the seller starts its actions in the market. This approach does not require special skills and experience and it is characterized by its simplicity. Furthermore, we can cover a lot of cases which can be faced in a dynamic environment because the developer only defines a set of number combinations for the input and output parameters and it is not obligated to define specific rules to cover all these cases.

B. Rules Generation

Fuzzy rule base consists of the main component of the FL controller because it results the value of the output parameter. In [10], authors provide a mathematical model and results for the deadline calculation based on crisp values for parameter α . In [11], authors present a set of 5 Fuzzy rules that result the value of α according to inputs for parameters q and ε . However, the definition of specific rules for the deadline calculation is a complex task because all the necessary aspects of the presented scenario should be covered. Hence, using an automatic methodology for rules extraction from a large set of values that cover complex aspects of the scenario seems to be more effective. Fuzzy rules induce a fuzzy partition of the product space of input and output variables. For this, fuzzy clustering techniques seem to be the appropriate tool for this partition detection. In this work, we use automatic generation methods from data given by experts, based on clustering techniques. We use two of the most widely used techniques: Subtractive [17] and Fuzzy C-Means clustering [18] & [19]. The main advantage of Subtractive clustering is that it can identify clusters centers without previous knowledge about their number. Additionally, Fuzzy C-Means can identify points that belong to two or more clusters providing a more fine-grained solution. We extend the study in [10] & [11] by the automatic generation of rules. Clustering is a very effective technique for grouping data in large data sets providing the relationship embedded in the data while FL is a very powerful tool for real time decision making under uncertainty.

In Subtractive clustering every data point is rescaled to [0,1] before its manipulation. For each of them, a potential degree P_i is defined according to its location to all other points. This potential depends on the Euclidean distance of the examined point to all the others. Moreover, the potential of each data point to be a cluster center is higher when more data points are closer to it. The point with the higher potential becomes the first cluster center and all the potentials for the other points are recalculated excluding the influence of the cluster center. The point with the highest potential becomes the next cluster center. The distance of the new candidate cluster center with the all previous defined cluster centers should fulfil a specific condition defined by the algorithm and ensures that cluster centers will have a minimum distance between them. If this condition is true then the point becomes the next cluster center or else it is rejected and its potential is set to 0. The potential degree for each point is calculated by:

$$P_j = \sum_{i=1}^N e^{-a \cdot \|x_i - x_j\|^2} \quad (4)$$

where

$$a = \frac{\gamma}{r_\alpha} \quad (5)$$

and x is the data point, N is the number of points, γ is a variable and r_α is the cluster radius. When a cluster center is

defined then the recalculation of all the other potentials is given by:

$$P_j = P_j - P_k \cdot \zeta \quad (6)$$

with

$$\zeta = e^{-b \cdot \|x_j - c_k\|^2}, \quad b = \frac{4}{r_b^2} \quad \text{and} \quad r_b = r_\alpha \cdot \eta \quad (7)$$

and P_k is the potential value data as a cluster center, c_k are clusters centers, b is the weight of j -data to cluster center, r_b defines the neighbourhood density measure and η is the quash factor. Values of each parameter are defined in the original article.

In Fuzzy C-Means algorithm, a point could belong to more than one clusters. The algorithm is based on the minimization of the following form:

$$J_k = \sum_{i=1}^M \sum_{j=1}^C u_{ij}^k \|x_i - c_j\|^2 \quad (8)$$

where M is the number of data points, C is the number of cluster centers, $1 \leq k \leq \infty$, u_{ij} is the membership degree of the x_i in the cluster j , x_i is the i^{th} measured data, and c_j is the center of each cluster. The membership degree is calculated by:

$$u_{ij} = \left(\sum_{m=1}^C \left(\frac{\|x_i - c_j\|^{\frac{2}{k-1}}}{\|x_i - c_m\|^{\frac{2}{k-1}}} \right) \right)^{-1} \quad (9)$$

and

$$c_j = \frac{\sum_{i=1}^M u_{ij}^k \cdot x_i}{\sum_{i=1}^M u_{ij}^k} \quad (10)$$

The number of inputs and outputs in the generated fuzzy system are as many as the number of the given data. The data are defined to reflect the policy of the seller described in the ‘Seller Behaviour’ Section. We have used a set of 90 data rows. Each row contains the values for parameters q , ε and α . Each input and output variable has as many membership functions as the number of clusters that are identified by algorithms. Moreover, the number of rules is as many as the number of clusters. In our case, using subtractive clustering, each input has 4 membership functions (Gaussian type) and the system has 4 rules. It should be noted that the cluster radius

was defined equal to 0.5. If we use a different value for cluster radius the number of rules changes as well as the number of membership functions (for example if we use the cluster radius equal to 0.3 we take 10 fuzzy rules). Using Fuzzy C-Means clustering, we also take, for the same data set, 4 membership functions and 4 fuzzy rules in our system. Each rule tries to map a cluster of the input space to a cluster of the output space.

V. RESULTS – TECHNIQUES COMPARISON

Table I shows a set of results concerning the deadline calculation for different values of parameters q and ϵ . For our experiments, we have used a top value for parameter α equal to 1000. In this Table, we list results taken from [10] and [11] for comparison purposes. Moreover, in Figures 2 and 3, we can see a graphical representation of the results. The most important advantage of the automatic methods for rules generation is that they provide a fast and efficient way to define the necessary rules. In other cases, developers should spend time and effort to define each rule trying to cover all the aspects of the problem.

Fuzzy approaches for the deadline’s calculation achieve more fine-grained results compared to [10]. The reason is that it is very difficult to define a crisp value for parameter α that will be valid for all the products. The value of α should vary

according to product’s characteristics. For this, using a crisp value for α results very small deadline in the majority of cases.

Moreover, when the intended profit is small (Figure 2a), we can discern that fuzzy techniques are in the same levels, concerning the deadline, while the subtractive technique gives larger deadlines. In the rest of the cases (Figure 2b and Figure 2c), we can see that the Fuzzy model proposed in [11] results larger deadline due to the fuzzy rule base defined by experts. However, automatic rules generation method seem to provide more fine-grained deadline values as the values of α are allocated in whole region of [0..1000] (especially in the case of using subtractive clustering).

In Figure 3a, we see that the clustering techniques lead to an intermediate deadline values while the fuzzy model in [11] provides large values and non-fuzzy model very small values. This is more efficient, because a deadline of 113 proposals is a large number even for a non-popular product. We should not forget that the seller spends resources at every interaction with buyers. Moreover, in Figure 3b, we see that clustering techniques lead to very small values for the deadline enhancing even more the policy of the seller which forces him to conclude quickly the BG when the product is very popular and the profit is a very large number.

TABLE I. SELLER’S DEADLINE CALCULATION.

Profit (ϵ)	Popularity Ranking measure q	T_s , for $\alpha = 50$ (from [10])	Fuzzy approach in [11]		Subtractive clustering technique		Fuzzy C-Means technique	
			α value	T_s	α value	T_s	α value	T_s
1	0.2	6	588	20	999	25	430	17
1	0.7	5	275	10	287	10	256	9
1	1	5	89.4	6	168	7	152	7
10	0.2	18	588	56	511	53	396	47
10	0.7	12	275	23	214	21	222	21
10	1	10	89.4	12	116	13	118	13
50	0.2	38	588	117	109	54	246	79
50	0.7	22	89.4	27	97.1	28	72	25
50	1	17	15.8	12	1	5	1	5
100	0.2	52	275	113	117	77	58.2	56
100	0.7	29	89.4	35	37.8	26	1	7
100	1	22	15.8	15	1	6	1	6

VI. CONCLUSIONS

In this work, we propose a FL-approach for the deadline calculation of a seller agent participating in a BG. We describe the behaviour of the seller which shows when the seller should stay more or less in the BG in order to gain as much profit as possible. The seller proposes prices for a specific number of rounds based on a certain policy. This policy refers to the patience exhibited by the seller. A patient seller should stay in the BG as long as possible. An impatient seller tries to sell the product as soon as possible by rapidly decreasing its prices.

The policy of the seller is implied by a scaling factor α . We describe a Fuzzy controller for the determination of the value of parameter α and we use two clustering methods for

the automatically rules generation. Moreover, we compare results taken from the model based on the automatic rules generation, concerning the final deadline of the seller, with results taken by previous studies such as in [10] and [11]. Rules based on data given by experts and clustering techniques lead to more fine-grained results comparing to a model that uses rules previously defined. Moreover, rules definition requires a lot of effort from the developers side, because all the significant aspects of the problem should be covered. Hence, through the automatic rules generation a short, efficient methodology for the deadline’s calculation is provided.

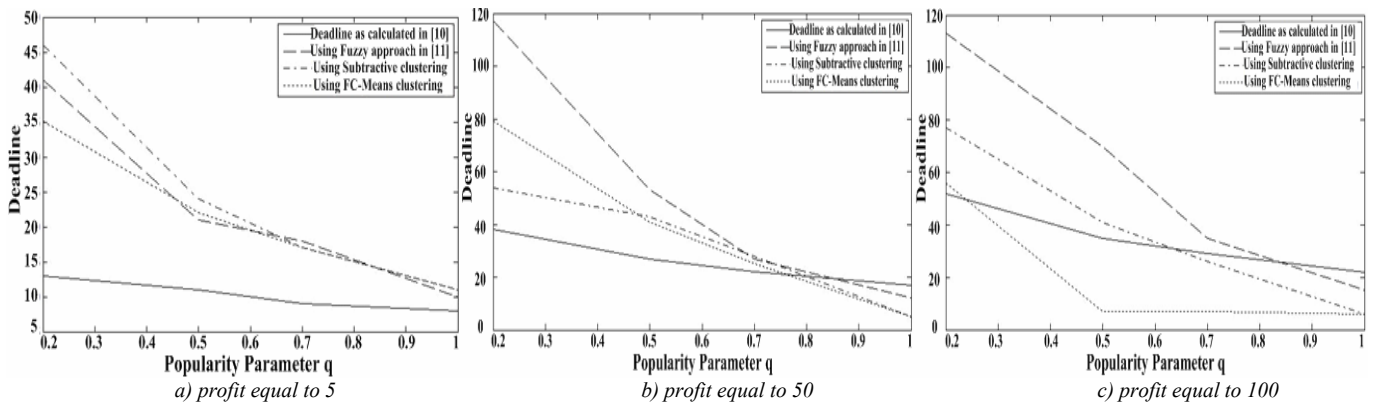


Figure 2. Graphical representations of the deadline vs popularity parameter for different values of profit.

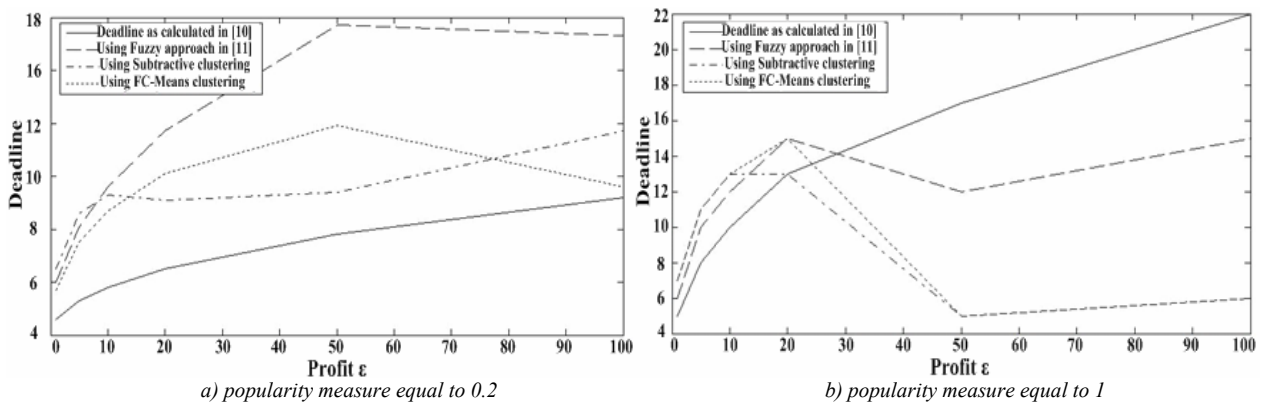


Figure 3. Graphical representations of the deadline vs profit for non-popular and popular products.

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