Non-cooperative Dynamic Spectrum Access for CDMA networks

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Abstract- Recent studies indicate the presence of a significant amount of idle licensed spectrum, in different time periods and geographic locations. Prompted by the latest regulatory changes and radio technology advances, dynamic spectrum access is becoming increasingly attractive, for alleviating the spectrum scarcity problem in the unlicensed bands. We study the dynamic spectrum access in a CDMA environment. We assume that users selfishly switch from one frequency band to another, so as to maximize their individual benefit, and model the studied problem as a non-cooperative game. We propose a probabilistic, distributed strategy with which users might share the available resources to satisfy their quality of service requirements, and minimize their transmitted power. The comparison of the proposed strategy against other alternative strategies shows that it combines fast convergence, efficiency and stability, while its simplicity makes it practical for a real network.

Keywords– Distributed frequency band selection, game theory, dynamic spectrum access, code division multiple access

I. INTRODUCTION

Until recently, radio devices were only allowed to operate in designated spectrum bands. Recent studies by the US Federal Communications Commission (FCC) have revealed that, even in major urban areas, only 30% of the licensed spectrum is being utilized on average. Hence, the use of opportunistic spectrum agile radios (OSAR) over assigned spectrum may open new directions for wireless, QoS-sensitive services that suffer from spectrum scarcity in the unlicensed bands. The FCC has issued a Notice of Public Rulemarking and Order concerning cognitive radio technologies [1]. Furthermore, the Defense Advanced Research Project Agency (DARPA) has launched the neXt Generation (XG) program for developing new adaptive mechanisms and technologies for spectrum sharing [2]. In both initiatives, spectral agile devices are capable of identifying and dynamically using the idle or sparsely-used frequency bands, thus, increasing the spectrum efficiency.

We study the dynamic spectrum access (DSA) problem for CDMA wireless networks. We assume that users have certain QoS requirements regarding their throughput, and are selfish, i.e., access the wireless resources according to their individual interests, without taking into account possible degradation to the QoS of the other users. Due to the selfishness of users, we use game theory for modeling their interactions and studying the equilibria of the system. We also propose a *distributed strategy for band selection* (DSBS) for efficient spectrum usage and stable operation in the discussed selfish environment, which can be easily adopted in a real network. The comparison of the DSBS against other strategies shows that it achieves fast convergence, while the equilibrium points are both efficient and stable.

The rest of the paper is structured as follows. We discuss prior related work, in Section II. In Section III, we describe our dynamic spectrum access system model for CDMA networks. In Section IV, we study the non-cooperative band selection game, and provide an appropriate distributed strategy for convergence to equilibria. In Section V, we discuss the performance of the proposed strategy. Section VI concludes the paper with our key findings and directions for future work.

II. PRIOR RELATED WORK

In [4], the cognitive radio architecture has been defined, and a prototype cognitive radio, named CR1, has been presented. In [5], continuous-time Markov models for dynamic spectrum access have been investigated, and have been shown to be accurate in predicting the dynamics of open spectrum access. In [6], the characteristics of opportunistic spectrum availability have been studied. However, none of the above works has modeled DSA for CDMA networks, with individual user QoS constraints, and, to the best of our knowledge, this issue has not yet been covered in the literature.

In [7], which is considered as the closest to our work, the problem of providing QoS for multimedia applications over OSAR TDMA networks has been investigated. A distributed, non-cooperative, channel-switching scheme has been proposed, and, similarly to our work, a game theoretic formulation has been adopted. One major difference from our work, as a result of the different access technique, is the adopted QoS model. In [7], the goal is to keep delay (due to collisions and retransmissions) under a certain value, and at the same time minimize the packet loss rate. In our model, due to CDMA, delay is not considered (lack of collisions). Instead, we are primarily concerned with the trade-off between the achieved throughput, and the required transmission power. Moreover, contrary to [7], we assume that all users may switch band at the same time, after the feedback broadcast by the serving BS. With this freedom, bestresponse strategies, as those proposed in [7], are rather unsuitable, as they typically lead to oscillations. For this reason, we propose an appropriate distributed strategy that may avoid such undesirable phenomena.

III. SYSTEM MODEL

In this section, we introduce our DSA model. We assume that a set of wireless terminals (WTs), $J = \{1, ..., N\}$, is competing for access to a set of available CDMA frequency bands, $C = \{1, ..., K\}$. We also assume the existence of a set of *base* stations (BSs), $I = \{1, ..., B\}$, and that each WT attaches to the BS with the strongest signal. WT *j* independently selects its transmission power p_i as well as its band c_i in order to maximize its "benefit".

In CDMA, the primary quantity that relates to the WT benefit is the signal-to-interference ratio (SIR) at the BS, as it is directly coupled with the achieved bit-error-rate, and, thus, throughput. We assume that each WT has certain QoS requirements in terms of lower and upper bounds on the achieved throughput, expressed by respective lower and upper bounds on its SIR, $\gamma_{m,j}$ and $\gamma_{M,j}$, respectively $(\gamma_{m,j} < \gamma_{M,j})^1$. The SIR γ_j of WT *j* transmitting in band *c* is

$$\gamma_j = \frac{W}{R} \frac{h_{a,j} p_j}{\sum_{\substack{k \neq j, c_k = c}} h_{a,k} p_k + \sigma_c^2}$$
(1)

W is the chip rate, and R the transmission rate (b/s), while the fraction W/R denotes the CDMA processing gain. $h_{a,k}$ denotes the path loss from WT k to BS a, and σ_c^2 the additive white Gaussian noise (AWGN) power at the BS in band c.

Observe that WT j may increase its SIR γ_i by increasing its transmission power p_i . However, by increasing p_i , the WT battery lifetime becomes shorter. In [3], a utility function quantifying the benefit of a WT as the number of bits successfully received per Joule of consumed energy (i.e., throughput versus energy consumption trade-off) has been introduced. In our model, apart from the transmission powers of the WTs, we extend the discussed utility function to take also into account their transmission bands.

$$u_j(\boldsymbol{p}, \boldsymbol{c}) = \frac{LR}{Mp_j} \left(1 - e^{-0.5\gamma_j} \right)^M \text{ (bits/Joule)} \quad (2)$$

M and L denote the length (bits) of a frame with and without headers, respectively. The term $(1 - e^{-0.5\gamma_j})^M$ denotes the probability of correct frame reception (for asynchronous FSK modulation) and is a measure of the achieved throughput.

In our model, the WTs, every T_c , select their transmission band, in response to band load information provided by the serving BS. Within a band selection period, the WTs adjust their transmission power, every T_p ($T_p < T_c$), based on interference information also provided by the BS. Due to the WT selfishness, we study the discussed situation as a combination of a power control game (PCG) and a band selection game (BSG), the former being *nested* into the latter.

The PCG has already been studied in [3]. It has been proven that it has a unique Nash equilibrium (NE) [9], i.e., a stable power allocation, where no WT can improve its utility by making individual changes to its transmitted power. At the discussed NE, $\partial u_i(\mathbf{p}, \mathbf{c})/\partial p_i = 0, \forall j \in J$ (first-order optimality condition), which is readily equivalent to

$$0.5M\gamma + 1 = e^{0.5\gamma}$$
. (3)

This means that at the NE all WTs enjoy the same SIR γ^* that solves (3). Note, however, that in the presence of SIR bounds, the following rule has to be applied for calculating the NE SIR [8]:

$$\gamma_{j}^{*} = \begin{cases} \gamma_{m,j}, \ \gamma^{*} < \gamma_{m,j} < \gamma_{M,j} \\ \gamma^{*}, \ \gamma_{m,j} < \gamma^{*} < \gamma_{M,j} \\ \gamma_{M,j}, \ \gamma_{m,j} < \gamma_{M,j} < \gamma^{*} \end{cases}$$
(4)

Note that each WT may independently solve (3) and, then, use (4), in order to determine its (target) NE SIR. Then, the WT, according to periodic feedback (total received interference) from the BS (every T_p), may adjust its transmission power to achieve its target SIR. After a finite number of successive power adjustments performed by all WTs, the NE power allocation is, finally, reached. The details of the discussed power control procedures fall outside the scope of this paper and will not be further analyzed. The interested reader may refer to [3], or [8] for more details. For the rest of this paper, it is sufficient for the reader to keep in mind that, for the period between two successive band updates, each WT transmits with its NE power p_i^* so as to achieve its NE SIR γ_i^* in the band it transmits.

Let us now analytically determine the NE power q_j^* (where $q_i^* = h_{a,i}p_i^*$ with which WT *j* reaches its BS, *a*, based on the linear system of SIR equations (see (1)):

$$q_{j}^{*} = \left(\frac{W}{R\gamma_{j}^{*}} + 1\right)^{-1} \cdot \frac{\rho_{c,a}^{2}}{1 - l_{c,a}}, \quad \forall j \in J \quad (5)$$

 $\rho_{c,a}^2$ denotes the other-cell interference received at BS a in band c (the interference component attributed to WTs attached to different BSs), i.e.,

(6)

$$\rho_{c,a}^{2} = \sum_{k \neq j, c_{k} = c, a_{k} \neq a} h_{a,k} p_{k} + \sigma_{c}^{2} ,$$

while

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$$l_{c,a} = \sum_{k=1,c_k=c,a_k=a}^{N} \left(\frac{W}{R\gamma_k^*} + 1\right)^{-1} .$$
 (7)

 $l_{c,a}$ is a measure of load on BS *a* in band *c* as a result of the target (NE) SIRs of the attached WTs. Note that, as more WTs attach to BS *a* in band *c*, the term $l_{c,a}$ increases, resulting in a corresponding increase of the NE power of the WTs. Note also that in order to have a feasible power allocation, the following condition must hold true:

$$l_{c,a} < 1$$
 (8)

Condition (8) represents the capacity limitation of the CDMA band, also termed *pole capacity*, beyond which the system is unable to settle to equilibrium.

¹ This is a valid assumption for delay intolerant, bandwidth-elastic services. where some variation in transmitted rate is not a problem but only very short delays are permitted (e.g., voice, video, real-time file transfers, etc).

IV. NON-COOPERATIVE DYNAMIC SPECTRUM ACCESS

In this section, we study the BSG. As already discussed, each WT, every T_c , selects its transmission band for the next period, based on load information provided by the serving BS. Over that period, it shares the selected band with other WTs. Mutually interfering WTs transmit with their NE power (see (5)), which depends on the other-cell inference $\rho_{c,a}^2$ (see (6)) as well as on the aggregate SIR requirements of the WTs (see (7)). Note, however, that from (3) and (4) the NE SIR, and, thus, throughput, of a WT does not depend on any of the aforementioned parameters. Hence, a WT may switch from one band to another and benefit by a power decrease, while maintaining its SIR (or throughput). Hence, in what regards the preference of a WT among a set of bands, only the transmission power is important, which means that utility function (2) can be simplified in the BSG. Specifically, it is sufficient to consider only the following quantity, which appears in (5):

$$Q_{c,a} = \frac{\rho_{c,a}^2}{1 - l_{c,a}} \qquad (9)$$

Observe that, if $Q_{c,a} < Q_{c',a}$, band *c* is preferable to band *c'* (for any WT), as the NE power will be lower in the former, compared to the latter band (while the achieved SIR will not be affected). Note, however, that if condition (8) is violated (i.e., $l_{c,a} \ge 1$), $Q_{c,a}$ assumes negative values (or may become infinite, if $l_{c,a} = 1$). This renders the $Q_{c,a}$ load metric unsuitable for use in the BSG². For this reason, we define an alternative metric, $L_{c,a}$, that approximates the mutual influence of the WTs attached to BS *a*, in band *c*:

$$L_{c,a} = \rho_{c,a}^2 \cdot l_{c,a} \quad (10)$$

Through the $L_{c,a}$ metric, a WT prefers band *c* to band *c'*, if $L_{c,a} < L_{c',a}$. A band allocation, in which no WT can unilaterally switch to another band with lower value of the $L_{c,a}$ metric, is an equilibrium, hereinafter referred to as *selfish band allocation* (SBA). Below we provide a formal definition of the SBA.

Definition 4.1 (BSG SBA): An outcome of the BSG is a SBA iff for every WT *j* that is attached to BS *a* and for all outcomes such that $c_k' = c_k$ and $c_j' \neq c_j$ the following inequality holds: $L_{c_{i},a} \leq L_{c'_{i},a}$.

The BSG, like similar settings, e.g., *dispersion games* (DG) [11], many settle to different equilibria that depend on the course of actions of the involved players, i.e., there is no unique equilibrium outcome.

A. Distributed Strategy for Dynamic Spectrum Access

As already discussed, WTs take decisions synchronously, in a distributed manner, based on updates on the status of the available bands broadcast regularly (every T_c) by the BS (see Section IV.B). We do not assume a sequential update scheme (as, for example, in [7]), as the considered environment is fully distributed and WTs are selfish; it is rather difficult for the BS to enforce WTs to follow a given band update sequence. This means that interactions among WTs can only be realized through the joint use of the shared bands.

Note that, if WTs employ best-response strategies, the system will typically suffer from oscillations (WTs forever hopping from one band to another), making it impossible to settle to a SBA [10]. It is, thus, important to introduce a strategy capable of avoiding such phenomena, by leading to SBAs, smoothly and rapidly.

We propose the *distributed probabilistic strategy for band selection* (DSBS) that is capable of leading the BSG to SBAs. One of the main characteristics of the DSBS is its simplicity, and that it can be easily realized in a real network.

Definition 4.2 (Distributed Strategy for Band Selection – DSBS): Given an outcome of the game, WT *j*, attached to BS *a* and transmitting in band *c*, using the DSBS will do the following:

- If $L_{c,a} \leq (F_a/K)$, select band *c* with probability 1,
- Otherwise, select band *c* with probability $v = (F_a/K)/L_{c,a}$ and with probability 1-*v* randomize over the frequency bands *c* 'for which
 - a) $L'_{c',a} < L_{c,a}$, and,

b)
$$l'_{c,a} < 1$$
,

where
$$F_a = \sum_{c=1}^{K} L_{c,a}$$
, $l'_{c,a} = l_{c,a} + \left(\frac{W}{R\gamma_j^*} + 1\right)^{-1}$, and
 $L'_{c,a} = \rho_{c,a}^2 \cdot l'_{c,a}$.

As described in Definition 4.2, according to the DSBS, a WT, if not satisfied by the load level in its current band (i.e., the load metric $L_{c,a}$ exceeds the balanced load, F_a/K), considers abandoning it. However, it does not always leave the band, but may stay in it with probability v, which is inversely proportional to the load level of the band. This ensures that not all WTs of a congested band abandon it. Specifically, with this probability, the *expected* remaining load in the discussed band, after all WTs have decided, will be the balanced load, F_a/K . Hence, at every round, the load distribution approaches the BSG SBA. Note also that when assessing (a) and (b), the WT takes into account its potential contribution to the respective conditions (i.e., load, and power allocation feasibility (8)), in order to avoid potential oscillations, and, finally, lead the BSG to the SBA. A formal proof on the convergence of the DSBS to SBAs is not provided here, for the sake of brevity. Convergence can be easily proven using ideas from analogous algorithms proposed for DGs.

B. A Simple Protocol for Distributed Band Selection

In the following paragraph, we introduce a simple protocol for providing band load feedback to WTs, so as to support the DSBS, or any other similar distributed band selection strategy. We assume that BSs are capable of scanning a considerable

² In an OSAR environment, the set of available bands may be capable of admitting all the WTs, if WTs are spread among the bands. However, if the majority of WTs is gathered in a restricted set of bands, the discussed bands might fail to provide the required capacity, i.e., (8) may not always hold true.

spectrum of frequencies and recognizing free spectrum bands³, i.e., bands that are not occupied by any primary user. BSs are also aware of the level resource saturation of the free bands, i.e., quantities $\rho_{c,a}^2$, and $l_{c,a}$ (see (6) and (7), respectively). Hence, BS *a* may compute for each band *c* a tuple of the form $\langle f_c, w_c, \rho_{c,a}^2, l_{c,a} \rangle$, where f_c and w_c denote the central frequency of the band, and its bandwidth⁴, respectively. The BS periodically (period T_c) broadcasts all gathered tuples in all frequency bands. Each WT may simply respond by switching to its "favorite" band.

If a BS discovers that one or more primary users started transmitting in a band, it notifies the secondary WTs to immediately abandon this band. Specifically, a notification message containing a list of *forbidden* bands is broadcast to all WTs. WTs that transmit in one of the forbidden bands switch directly to a *non-forbidden* band. With this simple feedback protocol, the BSs may enable distributed band selection performed by selfish WTs.

V. SIMULATIONS

In this section, we evaluate the performance of the DSBS versus other strategies with regards to the speed of convergence, as well as the "quality" of the resulting equilibria. Specifically, we compare our strategy with the *Freeze* [11] and the *elementary step system* (ESS) strategy [12]. Other candidate strategies that could be used include the *fictitious play learning rule* [13], and the *reinforcement learning* [14]. However, such strategies are rather unsuitable for the BSG, due to their complexity, not guaranteed convergence, and significant communication overhead [11].

TABLE I. THE LIST OF SIMULATION PARAMETERS

M, total number of bits per frame	80
L, number of information bits per frame	64
W, spread spectrum bandwidth	10 ⁶ Hz
R , bit rate	10^4 b/s
σ^2 , AWGN power at the receiver	5×10 ⁻¹⁵ W
modulation technique	non coherent FSK
p_{max} , maximum power constraint	2 W
N, number of WTs	1:45
$\gamma_{m,j}, \gamma_{M,j}$, SIR constraints of WT j	Uniformly distributed in
	the [1,20] interval
C, number of CDMA frequency bands	5
B , number of BSs	1

According to the *Freeze* strategy, at each round, the WT chooses a band at random. The WT "freezes", if, at some round, the load metric $L_{c,a}$ assumes lower or equal value to the balanced load level, F_a/K . From this point on, the WT always selects the same band regardless of the choices of the other WTs.

Contrary to Freeze and DSBS (where all WTs may play concurrently, after an update broadcast by the BS), with ESS, WTs take decisions one at each round, based on the bestresponse concept. The ESS has attractive theoretical properties (e.g., at each round the system "improves" while in concurrent distributed models, such as DSBS, the system might also "deteriorate"), but also has serious drawbacks [12]: 1) since only one WT may select a new band at each round, the convergence time is at least $\Omega(N)$, and 2) the centralized control of the order in which WTs make their updates is rather unattractive for a truly distributed system. The ESS is readily incompatible with our model, where WTs are free to select their band at any round. However, we compare the ESS to the DSBS, as it is often adopted, as in [7], due to its simplicity and predictable behavior.

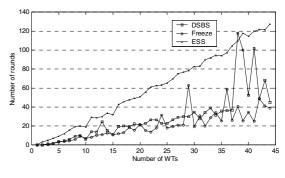
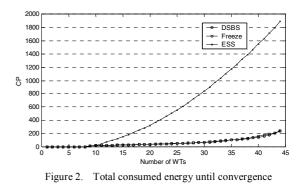


Figure 1. Number of rounds until convergence

Table 1 presents our simulation parameters. An important assessment metric is the number of rounds required for convergence. In Fig. 1, we observe that the Freeze achieves the fastest convergence. This is reasonable, since with Freeze a WT that finds itself in a "good" band does not consider leaving it in the next rounds ("freezes"). As the population of "frozen" WTs increases, the possible options for improvement of the remaining, "unfrozen", WTs rapidly decrease, thus, resulting to a quite fast convergence. We may also see that the DSBS converges almost as fast as the Freeze strategy. Finally, as anticipated, we observe the slow convergence speed of ESS.



Apart from the speed of convergence, it is also important to examine how energy-efficient the path from a given initial non-SBA state to a SBA is. This is quantified through the *CP* metric, which measures the aggregate consumed power until a

$$CP = \sum_{t=0}^{n-1} \sum_{j=1}^{N} q_j^*(t)$$
(11)

SBA is reached:

³ Such issues are outside the scope of this paper and, thus, not discussed here. The interested reader may refer to [15] for relevant details.

⁴ We assume that the considered bands are of equal bandwidth, for the sake of simplicity. However, the obtained results can be easily generalized for bands with different bandwidth.

n is the number of rounds until convergence, and $q_j^*(t)$ denotes the NE power with which WT *j* reaches the BS during round *t* (see (5)). Fig. 2 shows that, as anticipated, ESS is more power consuming than DSBS. A WT that operates on a very loaded band, by using ESS, will incur significant energy losses until its round to play and leave this band. We also observe that both Freeze and DSBS exhibit similar power efficiency.

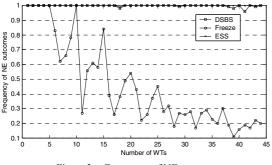


Figure 3. Frequency of NE outcomes

We also assess the "quality" of the resulting SBAs, as, after convergence, WTs remain static for a significant amount of time (until the arrival of a new WT, or the occupation of a band by a primary WT), which is typically much longer than the time to reach the SBA. An important issue is that a SBA may not always be a NE. This is due to the fact that we used (10) instead of (9), as the band load metric, for the reasons reported in Section IV.A. Hence, we examine whether with (10) the assessed strategies are capable of reaching a NE, i.e., a point where no WT may benefit by unilaterally selecting another band. In Fig. 3, we see that ESS and DSBS almost always achieve NE outcomes. On the other hand, Freeze is rather inefficient in reaching a NE, i.e., the achieved band allocations are rather unstable (after convergence, a WT by considering (9) may have the incentive to deviate).

From the discussion above, we may conclude that, even though the DSBS is a bit slower than the Freeze, the failure of the latter to converge to a stable band allocation (NE) is a major reason for preferring the former. Moreover, although the ESS guarantees stable outcomes, it is much slower than the DSBS, but more importantly is incompatible with our model where WTs are free to switch band at any round, and not in a predefined sequence. Hence, we believe that DSBS is a distributed strategy, which is suitable for use in a real system, as it is simple, fast converging, and stable.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have studied the problem of noncooperative, dynamic spectrum access in CDMA networks. We have considered an environment where WTs independently select their transmission band among a set of available CDMA bands, so as to maximize their personal benefit. We have modeled the BSG and proposed a distributed strategy, DSBS. We have assessed the performance of DSBS, against other strategies, showing that our strategy achieves fast convergence, while the achieved equilibria are both efficient and stable.

We plan to further assess the proposed strategy, using more complex and realistic simulation scenarios. We would also like to introduce new strategies based on the concept of bounded rationality, or learning algorithms from artificial intelligence.

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REFERENCES

- "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," the FCC Notice of Proposed Rulemaking and Order -ET Docket No. 03-108.
- [2] "http://www.darpa.mil/ato/programs/xg/rfc_vision.pdf,"
- [3] C. U. Saraydar, N. B. Mandayam and D. J. Goodman, "Efficient power control via pricing in wireless data networks", IEEE Transactions on Communications, Vol. 50, pp. 291-303, 2002
- [4] J. Mitola III, "Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio", Doctoral Dissertation, KTH, Stockholm, Sweden, May 2000.
- [5] Y. Xing, R. Chandramouli, S. Mangold and S. Shankar, "Dynamic Spectrum Access in Open Spectrum Wireless Networks", IEEE Journal on Selected Areas in Communications, Vol. 24, Issue 3, pp. 626-637, 2006.
- [6] X. Liu and W. Wang, "On the Characteristics of Spectrum-Agile Communication Networks", Proc. IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN), 2005.
- [7] A. Larcher, H. Sun, M. Van Der Schaar and Z. Ding, "Decentralized Transmission Strategies for Delay-sensitive Applications over Spectrum Agile Networks", Proc. Packet Video 2004, Dec. 2004.
- [8] G. Alyfantis, S. Hadjiefthymiades, and L. Merakos, "A Cooperative Uplink Power Control Scheme for Elastic Data Services in Wireless CDMA Systems", ACM SIGCOMM Computer Communication Review, Vol.36, No.3, 2006.
- [9] D. Fudenberg and J. Tirole, "Game Theory", MIT Press, Cambridge (MA), 1991.
- [10] W. B. Arthur, "Inductive Reasoning and Bounded Rationality (the El Farol Problem)", Amer. Econ. Review (Papers and Proceedings), pp. 406-411, 1994
- [11] T. Grenager, R. Powers and Y. Shoham, "Dispersion Games: General Definitions and Some Specific Learning Results", Proc. AAAI, 2002.
- [12] E. Even-Dar and Y. Mansour, "Fast Convergence of Selfish Rerouting", Proc. ACM-SIAM Symposium on Discrete Algorithms (SODA), 2005
- [13] G. Brown, "Iterative solution of games by fictitious play", in Activity Analysis of Production and Allocation, New York, John Wiley and Sons, 1951.
- [14] L. P Kaelbling, M. L. Littman and A. P Moore, "Reinforcement learning: A survey", Journal of Artificial Intelligence Research 4:237–285, 1996
- [15] H. Tang, "Some physical layer issues of wide-band cognitive radio systems", Proc. IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN), 2005.