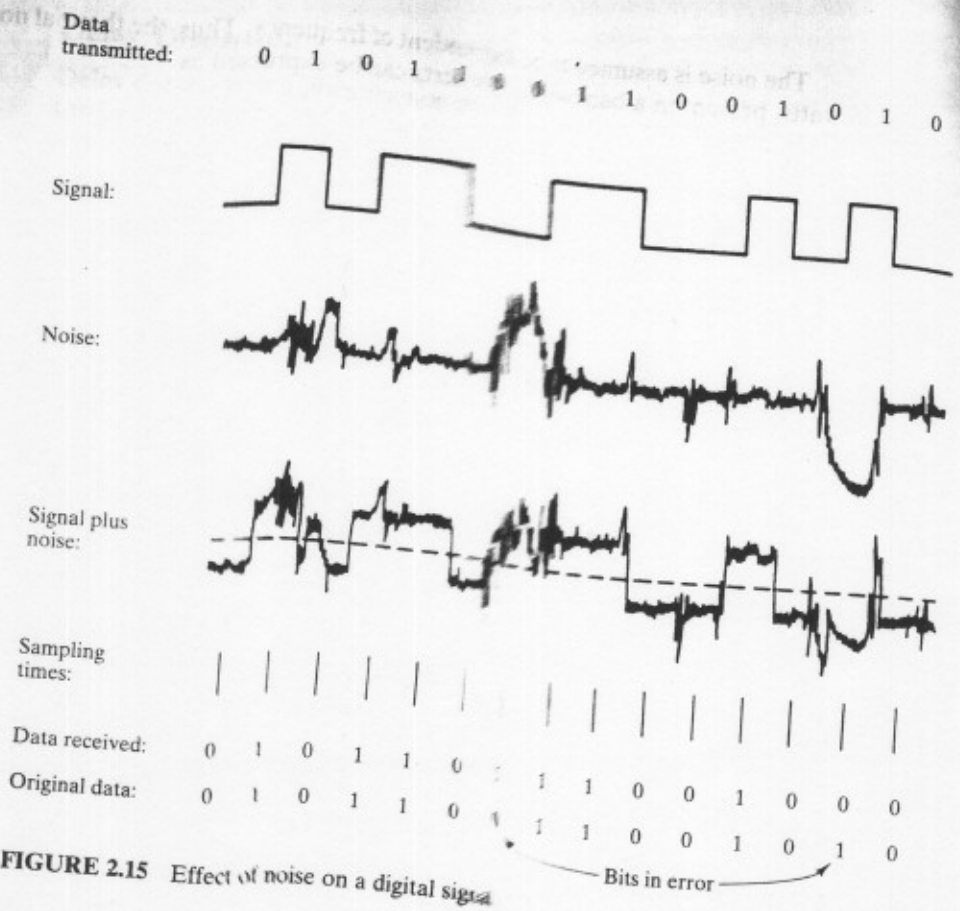


FIGURE 2.15 Effect of noise on a digital signal

the received waveform once per bit time. As can be seen, the noise is occasionally sufficient to change a 1 to a 0 or a 0 to a 1.

Channel Capacity

We have seen that there are a variety of impairments that distort or corrupt a signal. For digital data, the question that then arises is to what extent these impairments limit the data rate that can be achieved. The rate at which data can be transmitted over a given communication path, or channel, under given conditions, is referred to as the channel capacity.

There are four concepts here that we are trying to relate to one another:

- **Data rate.** This is the rate, in bits per second (bps), at which data can be communicated.
- **Bandwidth.** This is the bandwidth of the transmitted signal as constrained by the transmitter and by the nature of the transmission medium, expressed in cycles per second, or hertz.

- **Noise.** The average level of noise over the communications path.
- **Error rate.** The rate at which errors occur, where an error is the reception of a 1 when a 0 was transmitted, or the reception of a 0 when a 1 was transmitted.

The problem we are addressing is this: Communications facilities are expensive, and, in general, the greater the bandwidth of a facility, the greater the cost. Furthermore, all transmission channels of any practical interest are of limited bandwidth. The limitations arise from the physical properties of the transmission medium or from deliberate limitations at the transmitter on the bandwidth to prevent interference from other sources. Accordingly, we would like to make as efficient use as possible of a given bandwidth. For digital data, this means that we would like to get as high a data rate as possible at a particular limit of error rate for a given bandwidth. The main constraint on achieving this efficiency is noise.

To begin, let us consider the case of a channel that is noise-free. In this environment, the limitation on data rate is simply the bandwidth of the signal. A formulation of this limitation, due to Nyquist, states that if the rate of signal transmission is $2W$, then a signal with frequencies no greater than W is sufficient to carry the data rate. The converse is also true: Given a bandwidth of W, the highest signal rate that can be carried is $2W$. This limitation is due to the effect of intersymbol interference, such as is produced by delay distortion. The result is useful in the development of digital-to-analog encoding schemes and is derived in Appendix 4A.

Note that in the last paragraph, we referred to signal rate. If the signals to be transmitted are binary (two voltage levels), then the data rate that can be supported by W Hz is $2W$ bps. As an example, consider a voice channel being used, via modem, to transmit digital data. Assume a bandwidth of 3100 Hz. Then the capacity, C, of the channel is $2W = 6200$ bps. However, as we shall see in Chapter 4, signals with more than two levels can be used; that is, each signal element can represent more than one bit. For example, if four possible voltage levels are used as signals, then each signal element can represent two bits. With multilevel signaling, the Nyquist formulation becomes

$$C = 2W \log_2 M$$

where M is the number of discrete signal or voltage levels. Thus, for $M = 8$, a value used with some modems, C becomes 18,600 bps.

So, for a given bandwidth, the data rate can be increased by increasing the number of different signals. However, this places an increased burden on the receiver: Instead of distinguishing one of two possible signals during each signal time, it must distinguish one of M possible signals. Noise and other impairments on the transmission line will limit the practical value of M.

Thus, all other things being equal, doubling the bandwidth doubles the data rate. Now consider the relationship between data rate, noise, and error rate. This can be explained intuitively by again considering Figure 2.15. The presence of noise can corrupt one or more bits. If the data rate is increased, then the bits become "shorter" so that more bits are affected by a given pattern of noise. Thus, at a given noise level, the higher the data rate, the higher the error rate.

All of these concepts can be tied together neatly in a formula developed by the mathematician Claude Shannon. As we have just illustrated, the higher the data rate, the more damage that unwanted noise can do. For a given level of noise, we would expect that a greater signal strength would improve the ability to correctly receive data in the presence of noise. The key parameter involved in this reasoning is the signal-to-noise ratio (S/N), which is the ratio of the power in a signal to the power contained in the noise that is present at a particular point in the transmission. Typically, this ratio is measured at a receiver, as it is at this point that an attempt is made to process the signal and eliminate the unwanted noise. For convenience, this ratio is often reported in decibels:

$$(S/N)_{dB} = 10 \log \frac{\text{signal power}}{\text{noise power}}$$

This expresses the amount, in decibels, that the intended signal exceeds the noise level. A high S/N will mean a high-quality signal and a low number of required intermediate repeaters.

The signal-to-noise ratio is important in the transmission of digital data because it sets the upper bound on the achievable data rate. Shannon's result is that the maximum channel capacity, in bits per second, obeys the equation

$$C = W \log_2\left(1 + \frac{S}{N}\right)$$

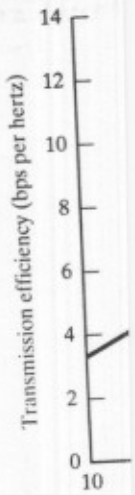
where C is the capacity of the channel in bits per second and W is the bandwidth of the channel in hertz. As an example, consider a voice channel being used, via modem, to transmit digital data. Assume a bandwidth of 3100 Hz. A typical value of S/N for a voice-grade line is 30 dB, or a ratio of 1000:1. Thus,

$$C = 3100 \log_2(1 + 1000)$$
$$= 30{,}894 \text{ bps}$$

This represents the theoretical maximum that can be achieved. In practice,however, only much lower rates are achieved. One reason for this is that the formula assumes white noise (thermal noise). Impulse noise is not accounted for, nor are attenuation or delay distortion.

The capacity indicated in the preceding equation is referred to as the error-free capacity. Shannon proved that if the actual information rate on a channel is less than the error-free capacity, then it is theoretically possible to use a suitable signal code to achieve error-free transmission through the channel. Shannon's theorem unfortunately does not suggest a means for finding such codes, but it does provide a yardstick by which the performance of practical communication schemes may be measured.

The measure of efficiency of a digital transmission is the ratio of C/W, which is the bps per hertz that is achieved. Figure 2.16 illustrates the theoretical efficiency



FIGURE

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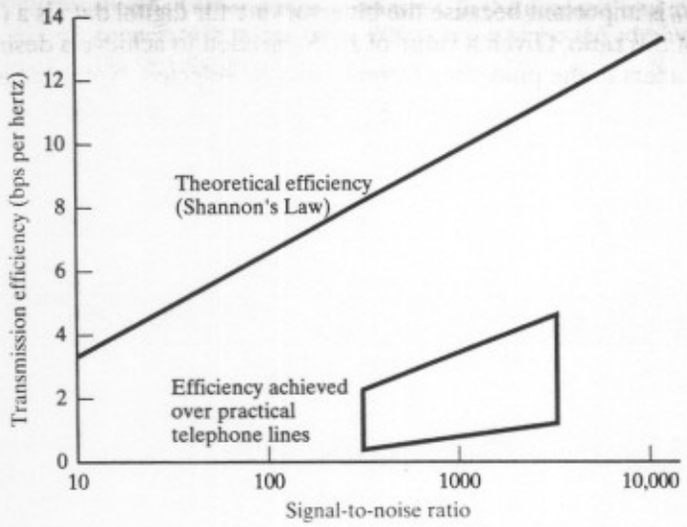

FIGURE 2.16 Theoretical and actual transmission efficiency.

of a transmission. It also shows the actual results obtained on a typical voice-grade line.

Several other observations concerning the above equation may be instructive. For a given level of noise, it would appear that the data rate could be increased by increasing either signal strength or bandwidth. However, as the signal strength increases, so do nonlinearities in the system, leading to an increase in intermodulation noise. Note also that, because noise is assumed to be white, the wider the bandwidth, the more noise is admitted to the system. Thus, as W increases, S/N decreases.

Finally, we mention a parameter related to S/N that is more convenient for determining digital data rates and error rates. The parameter is the ratio of signal energy per bit to noise-power density per hertz, E_b/N_0. Consider a signal, digital or analog, that contains binary digital data transmitted at a certain bit rate R. Recalling that 1 watt = 1 joule/s, the energy per bit in a signal is given by $E_b = ST_b$, where S is the signal power and T_b is the time required to send one bit. The data rate R is just $R = 1/T_b$. Thus,

$$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}$$

or, in decibel notation,

$$\frac{E_b}{N_0} = S - 10 \log R + 228.6 \text{ dBW} - 10 \log T$$

The ratio E_b/N_0 is important because the bit error rate for digital data is a (decreasing) function of this ratio. Given a value of E_b/N_0 needed to achieve a desired error rate, the parameters in the preceding formula may be selected. Note that as the bit rate R increases, the transmitted signal power, relative to noise, must increase to maintain the required E_b/N_0.

Let us try to grasp this result intuitively by considering again Figure 2.15. The signal here is digital, but the reasoning would be the same for an analog signal. In several instances, the noise is sufficient to alter the value of a bit. Now, if the data rate were doubled, the bits would be more tightly packed together, and the same passage of noise might destroy two bits. Thus, for constant signal and noise strength, an increase in data rate increases the error rate.

Example

For binary phase-shift keying (defined in Chapter 4), $E_b/N_0 = 8.4$ dB is required for a bit error rate of 10^{-4} (probability of error = 10^{-4}). If the effective noise temperature is 290°K (room temperature) and the data rate is 2400 bps, what received signal level is required?

We have

$$8.4 = S(\text{dBW}) - 10 \log 2400 + 228.6 \text{ dBW} - 10 \log 290$$

$$= S(\text{dBW}) - (10)(3.38) + 228.6 - (10)(2.46)$$

$$S = -161.8 \text{ dBW}$$

2.4 RECOMMENDED READING

There are many books that cover the fundamentals of analog and digital transmission. [COUC95] is quite thorough. Other excellent treatments include the three-volume [BELL90], [PROA94], and [HAYK94].

BELL90 Bellcore (Bell Communications Research). *Telecommunications Transmission Engineering, Third Edition*. Three volumes. 1990.

COUC95 Couch, L. *Modern Communication Systems: Principles and Applications*. Englewood Cliffs, NJ: Prentice Hall, 1994.

HAYK94 Haykin, S. *Communication Systems*. New York: Wiley, 1994.

PROA94 Proakis, J. and Salehi, M. *Communication Systems Engineering*. Englewood Cliffs, NJ: Prentice Hall, 1994.

2.5 PROBLEMS

2.1 **a.** For the multipoint configuration of Figure 2.1, only one device at a time can transmit. Why?

b. There are two methods of enforcing the rule that only one device can transmit. In the centralized method, one station is in control and can either transmit or allow a